DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE,

ELIZADE UNIVERSITY,

MTH 203 – LINEAR ALGEBRA I.

FIRST SEMESTER EXAMINATION 2019.

NSTRUCTION: ANSWER ANY FOUR. TIME: 2 HOURS .

Q1. (a) What do you understand by the term 'Vector Space.'

(b) Let V be the set of all ordered pairs of real numbers. If $a = (x_1, y_1)$ *and* $b = (x_2, y_2)$ are elements of V.

Write $a + b = (x_1 + x_2, y_1 + y_2)$, $\propto a = (0, \propto y_1)$, $0 = (0,0)$ *and* $-a = (-x_1, -y_1)$. Is V a vector space with respect to these definitions of linear operations? Give a detailed explanation of your answer.

Q2. (a) Define Linearly dependent and linearly independent set of vectors.

(b) Determine whether or not $\{V_1, V_2, V_3, V_4\}$ is linearly independent , where

$V_1 = (1,1,3)$,$V_2 = (1,3,1)$ $V_3 = (3,1,1)$ $V_4 = (3,3,3)$.

(c) What do you understand by the following terms ' Linear combination' and ' Linear Span ' ?

Express $U = (-1,2,0)$ as a linear combination of V_1 *and* V_2 given $V_1 = (1,2,3)$, $V_2 = (1,0,2)$

(d) Show that $v_1 = (1,1)$ *and* $v_2 = (2,1)$ span $\mathbb{R}^2$.

Q3. (a) Define the term 'linear transformation'.

For the following linear transformations $T: \mathbb{R}^n \longrightarrow \mathbb{R}^n$,

Find a matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^n$

(i) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$, $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ 3y \\ 4x + 5y \end{bmatrix}$

(ii) $T: : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ *satisfying* $T\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $T\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

(b) Determine whether the following function is a linear transformation

$T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ With $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}$ if not provide a counterexample to one of the properties.

Q4. (a) Suppose $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ is the linear transformation defined by

$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a \\ b+c \end{bmatrix}$ If P is the ordered basis $[p_1, p_2, p_3]$ and Q is the ordered basis $[q_1, q_2]$ where $p_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $p_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $p_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $q_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $q_2 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ what is the matrix representation of T with respect to P and Q.

(b) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be defined by $T(a_1, a_2, a_3) = (3a_1 + a_2, a_1 + a_3, a_1 - a_3)$

Consider the standard ordered basis $\{e_1, e_2, e_3\}$ with respect to this basis the coordinate vector of an element (a_1, a_2, a_3) is $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$. Find the matrix representation of T.

Q5. Let $\{e_i\}$ be the standard basis for $\mathbb{R}^3$, and consider the basis $f_1 = (1,1,1)$

$$f_2 = (1,1,0) \text{ and } f_3 = (1,0,0)$$

(a) Find the transition matrix P from $\{e_i\}$ to $\{f_i\}$.
(b) Find the transition matrix Q from $\{f_i\}$ to $\{e_i\}$
(c) Verify that $Q = P^{-1}$
(d) Show that $[V]_f = P^{-1}[V]_e$ for any $V \in \mathbb{R}^3$
(e) Define $T \in L(\mathbb{R}^3)$ by $T(x, y, z) = (2y + z, x - 4y, 3x)$

Show that $[T]_f = P^{-1}[T]_e P$.

Q6. (a) When is a linear transformation said to be Homomorphism and Isomorphism?

(b) Consider the space $V = \mathbb{R}^2$ with basis vectors $V_1 = (1,1)$ and $V_2 = (-1,0)$.

Let T be the linear operator on $\mathbb{R}^2$ define by $T(x, y) = (4x - 2y, 2x + y)$.

Find matrix of T relative to the given basis.

(c) Define **'BASIS'**.

Find a basis for the vector space V spanned by vectors $w_1 = (1,1,0)$, $w_2 = (0,1,1)$, $w_3(2,3,1)$ and $w_4 = (1,1,1)$.